Eq. (20b) can alternatively be written as

$$\psi(x) = \frac{\psi_0 \begin{matrix}\sin\\ \cos\end{matrix} \left\{ \int^x \beta(x) dx \right\}}{\sqrt{\beta(x)\alpha(x)}}. \tag{20c}$$

When differentiating the solutions (20b) and (20c), it is conventional[7] to consider that the denominator varies much more slowly than the numerator so that

$$\frac{\partial \psi(x)}{\partial x} = \frac{\pm j\psi_0 \sqrt{\beta(x)} \exp \left\{ \pm j \int^x \beta(x) dx \right\}}{\sqrt{\alpha(x)}}. \tag{20d}$$

[7] L. M. Brekhovskikh, "Waves in Layered Media," Academic Press, Inc., New York, N. Y., p. 196; 1960.

The solutions (20b) and (20c) are valid only when $\alpha(x)$ and $\beta(x)$ satisfy the condition

$$\left| \frac{\dfrac{\partial S_1(x)}{\partial x}}{\omega \dfrac{\partial S_0(x)}{\partial x}} \right| = \left| \frac{\dfrac{1}{\beta(x)} \dfrac{\partial \beta(x)}{\partial x} + \dfrac{1}{\alpha(x)} \dfrac{\partial \alpha(x)}{\partial x}}{2\beta(x)} \right| \ll 1. \tag{20e}$$

All of the solutions given in Sections II, III and IV must satisfy condition (20e).

Thin-Film Waveguide Bolometers for Multimode Power Measurement

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Summary—**Thin-film bolometers have been developed for measuring the total (unwanted) power that could be transmitted in any or all possible modes and at many frequencies above the normal operating band.**

The bolometer is a thin metal film which is placed so that it intercepts all the power flowing down the waveguide. When the power in the fundamental frequency is filtered out and only power at higher frequencies remains in the waveguide containing the bolometer, then it can be used to measure the total spurious power emitted by a high-power transmitter above its fundamental frequency band. Measurements have been made up to 15 Gc in *S*-band waveguide.

A variety of materials and shapes were tested and the bolometers were shown to be capable of measuring equally well several different modes and frequencies separately and in combination.

I. Introduction

A FIRST STEP in reducing RFI emission from a microwave transmitter is the accurate determination of the total power in all the undesired frequency components traveling in the waveguide transmission line. A power meter that operates over an extremely broad band and maintains uniform sensitivity for all modes that might exist within the measurement band would be a very useful RFI monitoring device. This conclusion was reached after examining the facts and arguing from them as follows.

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A harmonic sampler [1] had been designed earlier at Stanford Research Institute, and further developed at Airborne Instruments Laboratory, Deer Park, N. Y. [2]. Such a device has been used to measure the spectral output of a high-power source from the second to the sixth harmonic [3]. This versatile instrument is a relatively complicated device. It was felt that a simpler instrument, which would measure only the *total* spurious output from a high-power source (without giving the spectral distribution), would be a most useful adjunct to a high-power system when it is required to minimize the spurious-frequency output. This conclusion was based partly on the following measured result [3]: In the process of adjusting the electrode voltages of a high-power klystron, it was found that minimizing the second harmonic tended to minimize all the other harmonics also. It will of course require more measurements to determine whether this result ordinarily holds. In the meantime, however, it is suggested that a measurement of the single quantity, the *total spurious power* (without regard to spectral distribution), may enable one to set the control voltages to minimize any particular RFI. (Furthermore it might also indicate the approximate power level at any particular frequency, if the spectral distribution is known beforehand.) The reader should again be cautioned that this supposition is based on but a single measurement [3], [4] and that it will have to be tested much more extensively before it can be used with any degree of confidence. At the very least, it will be useful to know the total spurious output of a high-power transmitter as an indication of its RFI potential.

This paper describes work done on developing a thin-film bolometer with dimensions equal to, or greater than, the transverse cross section of a waveguide. In practice such a device would be used to measure the power in a waveguide from which the fundamental frequency output had been filtered out, for instance, as indicated schematically in Figs. 1 or 2. A 3-db directional coupler is suggested in Fig. 1. This operates on the same principle as the first 3-db coupler of a blanced duplexer with the two TR-cells fired. The input is equally divided into two waveguides which reflect the fundamental frequency band with such a phase as to cause them to combine (ideally without loss) and emerge from the fourth waveguide (Fig. 1) without reflection into the input. The harmonics, on the other hand, pass on to be measured, except for such reflections as are inherent in the 3-db coupler at the higher frequencies. The scheme suggested in Fig. 2 would make use of a 0-db directional coupler used in the manner of a harmonic pad [4]–[7]. A 0-db coupler, or crossover coupler, can be designed in many ways; perhaps the simplest form of 0-dg coupler is a cascade of two 3-db couplers, as in a balanced duplexer with the two TR-cells not fired, or simply removed. The essential idea is that the device acts as a 0-db coupler only in the fundamental frequency band, causing these frequency components in the input to cross over, while the harmonic frequency components in the input tend to go straight through (Fig. 2), and are thus separated from the fundamental frequency band. Not shown in Fig. 2 is a high-pass filter which could be inserted between the 0-db coupler and the thin-film bolometer to ensure that the fundamental frequency reaching the bolometer was well below the higher spurious frequencies. (For instance, a waveguide of reduced width could be used as a high-pass filter, as was indicated in Fig. 1.)

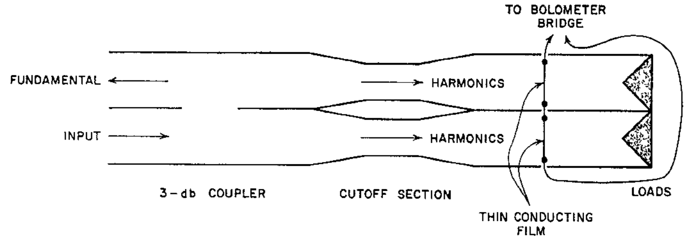


Fig. 1—3-db directional coupler and thin-film bolometer.

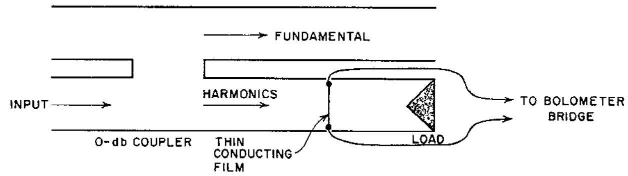
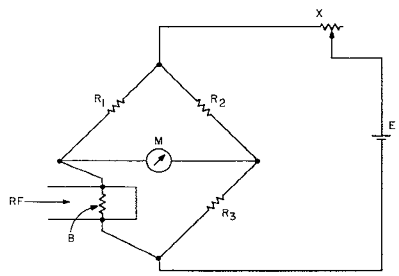


Fig. 2—0-db directional coupler and thin-film bolometer.



B THIN-FILM BOLOMETER IN WAVEGUIDE
R_1, R_2, R_3 RESISTORS WHOSE RESISTANCES SHOULD NOT CHANGE WITH TEMPERATURE
M SENSITIVE DETECTOR OF UNBALANCE CURRENT
X VARIABLE RESISTOR TO CONTROL DC (or A C) POWER INTO 'B' TO RESTORE BRIDGE BALANCE ON 'M'
E BATTERY (or A C SOURCE).

Fig. 3—Bridge circuit for balancing bolometer resistance.

The ideal bolometer and its mount should absorb either all or some constant fraction of the incident power. It should be well matched over a wide frequency range, either intrinsically or with the aid of a suitable absorbing load, in which case it should attenuate uniformly all frequencies of interest. If the bolometer absorbed only a portion of the incident power, this factor would be included in the calibration of the measuring setup. In a typical arrangement, a bolometer forms one resistive arm of a balanced bridge (Fig. 3). Since the bolometer has a considerably larger temperature coefficient of resistance than the resistances in the other three arms, the bridge is initially brought to balance by means of dc (or low-frequency ac) bias current. Microwave power then heats the bolometer and unbalances the bridge. The bridge is brought into balance again by removing an equivalent amount of dc (or low-frequency) power, *i.e.*, by decreasing the bias current. The microwave power is then calculated from the current change and bridge resistance, or displayed automatically in self-balancing instruments.

Thin-film and wire bolometers of various forms have been used in the past as single-mode waveguide power-measuring devices. One of the earlier of the film-type bolometers, the enthrakometer [8], samples the power passing through a waveguide by measuring the heating effect of the wall currents on two films which replace sections of the broad and narrow waveguide walls. This scheme in general is satisfactory for principal-mode transmission within a prescribed frequency range, but fails when it is necessary to measure the power over a very broad band in a great many possible modes. This is perhaps best understood if one considers the most extreme case imaginable, namely, the projection of a parallel beam of light through a pipe. If the pipe diameter is sufficiently larger than the beam diameter and the beam is nondivergent (as is, for example, a laser beam) then the currents induced in the pipe walls are negligible compared with currents induced in a waveguide with the same power flow at microwave frequencies. The essential point is that electromagnetic power flows through the waveguide cross-sectional area and the bolometer film should therefore completely cover this area to intercept the total power flow.

Thin-film bolometers intended for microwave power measurements have been developed to be inserted across a waveguide [9]–[12] or coaxial line [13]. These previous analyses and investigations were also only concerned with the measurement of power transmitted in a single mode. In the waveguide [9]–[12] case the film was a narrow transverse strip in the center of the waveguide and did not cover the full cross section of the waveguide; it was thus inherently insensitive, for example, to power in the TE_{20} mode.

A film bolometer that covers the full waveguide cross section will intercept the flow of power no matter what the mode or frequency. In the limit, for many modes and frequencies, the integrated electric field squared is an accurate indicator of the total power flow, and a partially or a totally absorbing thin-film bolometer covering the full waveguide cross section should be intrinsically superior to other bolometer forms.[1] The ability of the film bolometer to measure the total power in two modes and at several frequencies was shown by the experiments described herein. Also, single-mode power measurements were made for three different waveguide modes.

II. Materials and Methods

A variety of different conductive film materials and substrates was used in experiments to devise a stable, sensitive bolometer film for *S*-band waveguide (2.840 by 1.340 inches ID). The films were made by evaporating gold, silver, a gold and germanium mixture, nichrome or lead telluride on to thin mica sheets or Corning glass sheets 0.002-inch thick. Also, tracing paper painted with different mixtures of a silver paint,[2] conductive carbon,[3] and RF resistance cards were tested. Each film or sheet had a narrow, highly conductive, silver strip painted on its two broad or on its two narrow (parallel) edges, to which wire leads were attached. Fig. 4 shows some sample films: the upper left film is silver paint on tracing paper; the upper right film is gold on mica; the two center films are gold on glass, mounted on (but insulated from) metal frames; and the lower film is gold on mica in the shape of a rhombus. The rectangular films were mounted as shown in Fig. 5, while the rhombic film was mounted as shown in Fig. 6. Both film shapes intercept (but do not necessarily absorb) all

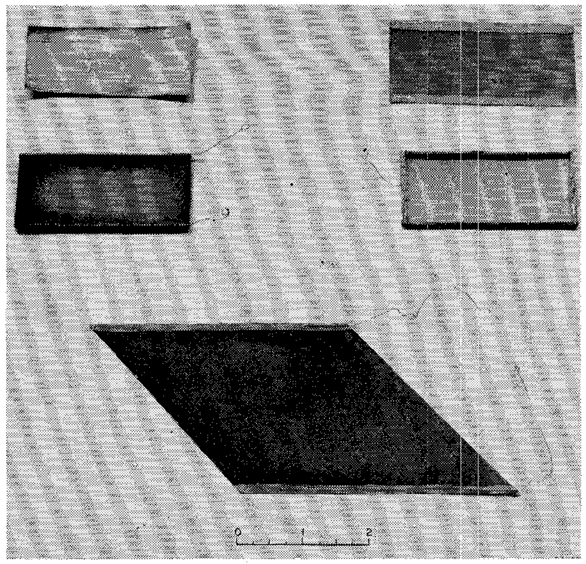


Fig. 4—Film bolometers of several shapes and materials.

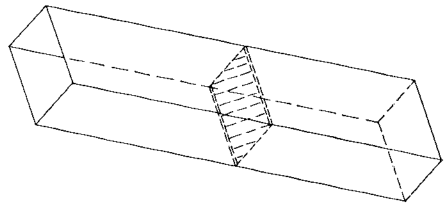


Fig. 5—Transverse film bolometer.

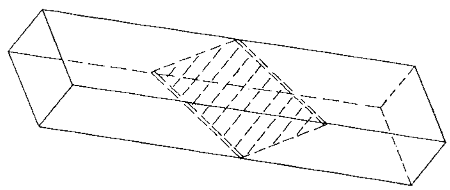


Fig. 6—Rhombic film bolometer.

of the power flowing in the waveguide. The rhombic film is potentially capable of absorbing nearly all the power independent of the waveguide termination, whereas the tranverse rectangular film backed by a matched load can at best absorb half the incident power. (In the case of a short-circuit plate placed behind a transverse film, a perfect match is theoretically possible at certain frequencies and modes, for which the short circuit is one-quarter guide wavelength behind the film.)

The conductive film was insulated from the waveguide in each case. However, there was enough capacitive coupling between the silver contact strips and the broad waveguide walls (or mounting cell), and to some degree between the short edge of the film (where there was no silver strip) and the narrow waveguide walls, for the film to act essentially as a shunt conductance across the waveguide in each transverse direction.

At first the dc film resistance was made, as nearly as possible, either 100 or 200 ohms (corresponding to 200 or 400 ohms per square) in order to use the bolometer

[1] This statement is predicated on the assumption that no substantial power is carried by modes near cutoff where the wave impedance is either very high (TE modes) or very low (TM modes). In the first place, it is unlikely that much power is carried in such modes because it would be difficult to couple into them efficiently. In the second place the impedances could all be brought closer, if necessary, by increasing the waveguide transverse dimensions [2].

[2] DuPont Silver Preparation, Electronic Grade 4817.

[3] Micro-Circuits Company Medium Resistance Paint R21.

with a commercially available power meter (Hewlett-Packard Model No. 430 C), designed to be operated with a bolometer of either 100- or 200-ohm dc resistance. The film thickness as such was not measured or controlled. The dc film resistance was found to be difficult to control precisely in manufacture. Since the bolometer resistance never exactly matched the 100- or 200-ohm bridge-arm resistance, an additional external resistance, either in series or in shunt with the film (as appropriate), was used to balance the power meter bridge. This complication appeared to be a source of trouble; the problem was finally solved by using a manually operated dc power-substitution bridge (Fig. 3) constructed in the laboratory. The dc bridge was found to be more accurate than the commercial power meter for this application. The bridge arms consisted of small adjustable wire-wound power resistors. When it was found that the movable tap on these resistors did not give sufficiently fine adjustment to balance the bridge easily, wire-wound potentiometers were added. Film resistances used in the dc bridge were of the order of 200 ohms for transverse films. (The optimum film resistance from the standpoint of maximum power transfer[4] to the film bolometer is a resistance equal to one-half the waveguide characteristic impedance, which however varies with mode and frequency.) In all cases transfer of power to the bolometer was calculated by measuring or calculating the power transmitted to the load (if any) behind the bolometer. (The net power transmitted past the bolometer is, of course, zero when a short circuit, instead of a load, is used behind the bolometer.)

Of all the conductive materials tested, the gold film was found to be the most sensitive, no doubt because of its relatively high temperature coefficient of resistance, and the nichrome film the least sensitive. Carbon films exhibited a negative temperature coefficient and were found to be quite unstable. The gold-germanium mixture appeared at certain times to be quite unstable —in fact, none of the films was as stable as commercially available wire bolometers. There are other metals that might be suitable. For instance platinum has been used extensively. It has approximately the same temperature coefficient of resistivity as gold (0.003/°C) but melts at a considerably higher temperature (1774°C for platinum vs 1063°C for gold) and is therefore more difficult to evaporate.

A bolometer mount with a transverse rectangular film is shown in Fig. 7 (exploded view). Electrical (dc) contact to the silver strips on the film is made directly by the separate split flange which is insulated from the waveguide flanges on either side. No microwave power leakage through the noncontacting flanges was detected.

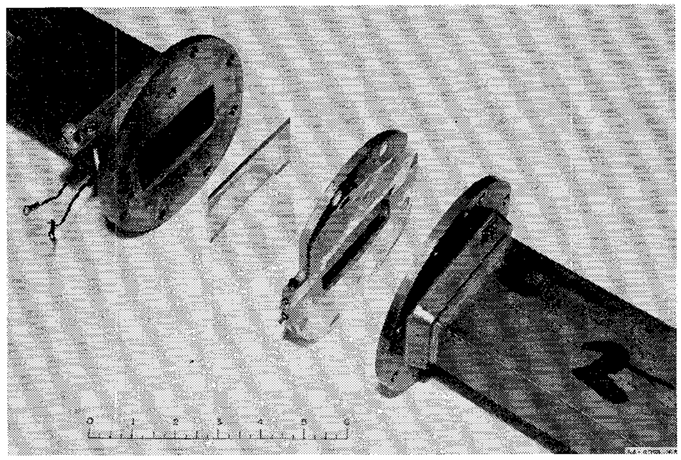


Fig. 7—Split-ring film bolometer mount (exploded view).

Tests were performed on *S*-band waveguide film bolometers at frequencies from *S* through *X* band and for the TE_{10}, TE_{01} and TE_{20} modes, both for single frequencies and for combinations of two frequencies in cross polarized modes. To ensure that mode purity was maintained in each separate measurement, the input waveguide was excited by a coaxial-to-waveguide transition operating in its single-(TE_{10}) mode region and this power was fed into the thin-film bolometer through long tapered waveguides.

To launch the TE_{20} mode the incident power was first divided equally and in phase at a coaxial tee junction, and then fed out of phase into two waveguides (by inverting the two coaxial-to-waveguide transitions with respect to one another); the two waveguides now carrying two out-of-phase TE_{10} modes were then combined in a Y junction to produce the TE_{20} mode.

The scheme used for feeding the power at two frequencies in different modes into a single film is shown in Fig. 8. Single-mode power impinges on the film from each side through septums designed to pass only one mode. These same septums totally reflect power from the other mode that passes through the other septum and the film. Thus, it is possible to calculate the power absorbed by the film from each mode present from the net forward power flow in that mode (measured separately for each mode by means of a conventional laboratory power meter) minus the reflected power (calculated from the measured VSWR of that mode). The total calculated absorbed power (for all modes present) is then compared with that indicated by the thin-film bolometer bridge measurement when both modes are excited at the same time. The septums consisted of movable polyfoam waveguide inserts with slots for metal foil sheets as shown in Fig. 9. In practice, the

[4] For transverse films backed by a matched load, maximum power is transferred to the bolometer at 3.0 VSWR, because the transverse film is an almost pure shunt conductance; this result follows from the fact that the film should ben be matched to the generator and load admittances in parallel, and should therefore have a normalized conductance of 2. Thus the load and film together present a conductance of 3 to the generator, which gives rise to a VSWR of 3:1. One-half of the available power is then delivered to the film, while one-quarter is reflected, and one-quarter is transmitted to the load.

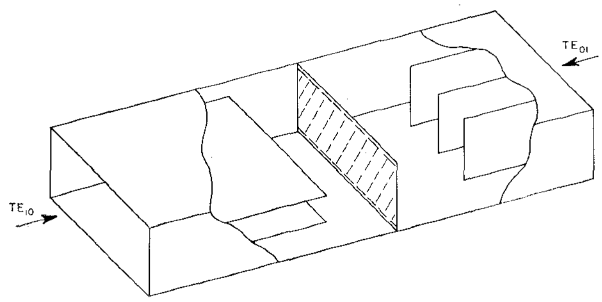


Fig. 8—Transverse film bolometer with septums for frequency and mode mixing.

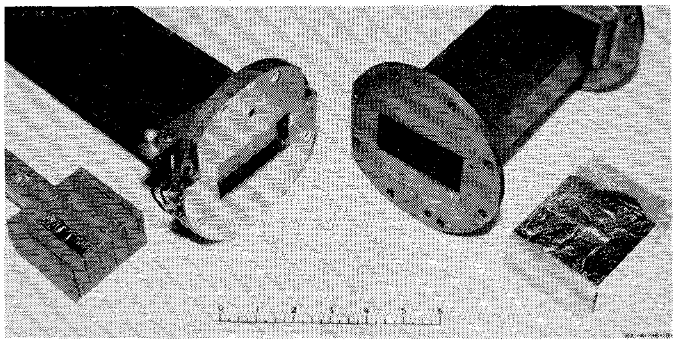


Fig. 9—Bolometer mount as sketched in Fig. 8.

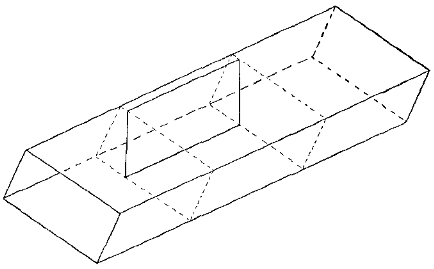


Fig. 10—Rectangular film bolometer mounted longitudinally and inclined to the waveguide walls.

inserts were set for minimum VSWR (maximum power absorption) in each case.

It was mentioned above[4] that a transverse film in shunt with the waveguide and backed by a matched load can absorb, at most, half the available power, and one quarter then goes into the matched load. Thus, the VSWR seen by the generator when maximum power is absorbed by the film is 3:1. It is possible in this case to obtain a better VSWR (and, incidentally, higher power transfer), using films that are not transverse. For instance, the rhombic film in the lower part of Fig. 4, and mounted as shown in Fig. 6, could be made to have a lower VSWR, but it was found later to measure power less accurately. This was attributed largely to its nonrectangular shape (see Section IV).

Tests were then made with *rectangular* films mounted in other than transverse positions, as indicated in Fig. 10. Although good results were obtained with this method of mounting, one may expect the power leaking past the film (which is not absorbed by the film) to increase with increasing frequency. For this reason this method of mounting the bolometer film (Fig. 10) is not as efficient as the method in which the bolometer film completely traverses the waveguide cross section (Figs. 5 and 7).

III. Power Measurements with Thin-Film Bolometers

The principal results of a large number of microwave power measurements on various thin-film bolometers are presented in Tables I through III. In each case the reflected power (calculated from the VSWR) and the measured power transmitted past the thin-film bolometer into a matched load (if any) behind the bolometer were subtracted from the measured incident power; this difference power is presumably absorbed in the thin film. The value of power thus obtained was then compared with the dc substitution power required to balance the bridge. The latter is, of course, the thin-film bolometer power measurement. The two values of power should be equal. The accuracy is limited by 1) a systematic (or average) error (which is probably due to some fundamental limitations of the film and circuit, and is less in the situations described in Tables I and III) and 2) by a random error (which is at least in part an experimental measurement error, and includes all sources of experimental error). A positive error means the thin-film bolometer power measurement is greater than the power calculated from the measurements made by standard methods, and vice versa. Where more than one mode was used, the power absorbed by the film was measured for each mode separately and the sum was compared with the thin-film bolometer measurement.

The first series of measurements was made with a rectangular film mounted transversely in the waveguide (Fig. 5). The film was made one arm of the bridge of a commercial laboratory-type microwave power meter (Hewlett-Packard Model No. 430 C), which substituted ac power at about 10 kc and was self-balancing. The thin-film bolometer merely replaced the usual coaxially mounted wire bolometer. Both TE_{10} and TE_{01} measurements were made in the range from 2.6 to 15.0 Gc. A total of 26 measurements (19 on the TE_{10} mode and 7 on the TE_{01} mode using three films of 130-, 180- and 230-ohms resistance) were made in this first series. A matched load backed up the film in each case. The average error was found to be -5.9 db, with about 1.7-db standard deviation.

The measurements shown in Tables I to III were made with a dc substitution bridge, and were found in general to have a smaller average error than measurements made with the direct reading power meter. The standard deviations of the errors are about the same in both cases, which indicates that the two methods of instrumentation can give equally accurate results,

TABLE I

AVERAGE ERROR OF THIN-FILM BOLOMETERS USED WITH DC BRIDGE—RECTANGULAR FILMS AS SHOWN IN FIGS. 5 AND 8

Line No.	Resistance	Waveguide Mode	Number of Measure-ments	Frequency Range Gc	Type of Back-Up Termination	Average Error* db	Standard De-viation db	Comments
1	165	TE_{10} TE_{01}	11 2	2.6–9.1	Matched load	−0.7	0.8	Combined results of two films
2	165	TE_{10} TE_{01}	8 8	3.0–4.0 6.9–7.0	Short circuit (septums)	−0.44	0.8	All measurements taken separately compare below
3	165	TE_{10} TE_{01}	8	3.0–4.0 6.9–7.0	Short circuit (septums)	−0.85	0.4	Film exposed to 2 modes at different frequencies simultaneously
4	165	TE_{20}	2	7.0–9.5	Matched load	+1.9	—	
5	165	TE_{20}	2	7.0–9.5	Short circuit	+0.4	—	

* A negative sign means that less dc power than RF power was required to rebalance the bridge; a positive sign means that more dc power than RF power was required to rebalance the bridge. With a negative sign, RF power is apparently "lost" somewhere. A positive sign could arise if the other resistances in the bridge are also temperature-sensitive.

TABLE II

AVERAGE ERROR OF THIN-FILM BOLOMETER USED WITH DC BRIDGE—RHOMBIC FILMS AS SHOWN IN FIG. 6

Line No.	Film Re-sistance Ohms	Waveguide Mode	Number of Measure-ments	Frequency Range Gc	Type of Back-Up Termination	Average Error* db	Standard De-viation db	Comments
1	95	TE_{10}	6	3.0–7.1	Short circuit	−4.5	0.7	
2	135	TE_{10}	7	3.0–7.1	Short circuit	−2.8	(See text)	Error strongly correlated with frequency
3	95	TE_{10}	7	3.0–9.5	Matched load	−4.3	0.8	
4	95	TE_{01}	2	7.1–9.5	Matched load	−1.5	0.5	

* See Footnote to Table I.

TABLE III

AVERAGE ERROR OF THIN-FILM BOLOMETERS USED WITH DC BRIDGE—RECTANGULAR FILMS PLACED LONGITUDINALLY IN WAVEGUIDE AS SHOWN IN FIG. 10

Line No.	Position of Film	Film Resistance	Waveguide Mode	Number of Measure-ments	Frequency Range Gc	Type of Back-Up Termina-tion	Average Error* db	Standing Deviation db	Com-ments
1	At angle near narrow wall	165	TE_{10} TE_{01}	7 5	3.0–12.0	Matched load	+0.2	1.3	
2	At angle centered in W.G.	165	TE_{20}	2	7.0– 9.5	Matched load	−0.1	0.8	
3	At angle centered in W.G.	165	TE_{20}	1	7.0	Short circuit	−1.3	—	

* See Footnote to Table I.

provided the average or regular errors are known. The measurements recorded in Table I were made on a rectangular film mounted transversely in the waveguide, as in the previous case.

Also shown in Table I (second and third lines) are measurements on the TE_{10} and TE_{01} modes, impinging either separately (line 2) or together (line 3) on the film according to the scheme of Fig. 7. The average error of 16 separate measurements prior to mixing the modes was −0.44 db, and the error when the modes were mixed was −0.85 db. In each case the reference level is the calculated total input power to the film bolometer as previously explained. Thus, there is an increase of only about −0.4 db in the measurement error when two modes at different frequencies are absorbed simultaneously by the thin film, and the dc substitution power is compared with the sum of the two RF powers; this increase is only of the same order as the measurement accuracy. On lines 4 and 5 of Table I are measurements for the TE_{20} mode. The average error is positive in these cases (unlike most of the other measurements), but this is probably not significant statistically because of the small number of measurements made.

In Table II are shown the results of measurements on a rhombus-shaped film of the type shown at the bottom in Fig. 4. The average errors recorded for the rhombus-shaped film are noticeably larger than the average errors of measurements made with rectangular films, whether transverse or longitudinal (see Table III). It is believed that the rhombic film is inherently less accurate because of its nonuniform dc current distribution. For the case of line 2 in Table II, the error appeared to be strongly correlated with frequency (the correlation is not evident in the table), so that the standard deviation was not calculated in this case.

The rectangular thin-film bolometers originally designed to be placed transversely in the waveguide were also tested in various longitudinal orientations (one of which is shown in Fig. 10). These longitudinal orientations have a disadvantage in that attenuation of the microwave power tends to decrease slowly as frequency increases. Table III gives the results of measuring a rectangular film placed as shown in Fig. 10. In one case the film was in the middle of the waveguide, in another the film was close to a narrow wall almost touching the wall at one edge. With this arrangement the film can attenuate and measure accurately the power in all modes below some upper frequency limit.

All single-frequency, single-mode measurements that were taken with the aid of the dc bridge (Tables I, II and III) were finally sorted out according to mode and were consolidated with the following results. The average error for the TE_{10} mode was -1.8 db[5] (46 measurements). The average error for the TE_{01} mode was -1.0 db[5] (16 measurements). The average error for the TE_{20} mode was $+0.5$ db[5] (7 measurements). Taken together, these results indicate that the thin-film bolometer in all three mounting positions probably does not discriminate strongly between different modes.

Not much has been said heretofore on the VSWR measurements because their effects were accounted for and it is not a basic problem. The range of VSWR was typically under 1.7 for longitudinally placed rectangular films and rhombic films, backed up by a matched load; some films showed VSWR's much better than that. Transverse films typically showed less than 3.2 VSWR with a matched load behind the film.[5]

IV. Analysis of a Thin-Film Bolometer

In order to better understand the action of this type of thin-film bolometer, a simplified analysis was made. Four possible elementary hypothetical states of a rectangular film with respect to microwave and dc power distribution were considered as shown in Fig. 11. Analogies were then drawn between these cases and actual films. Because of the large surface area-to-perimeter ratio of these films, nonuniform cooling by conduction was considered to be of minor importance and local film temperature was assumed to be a linear function of power dissipated locally. In effect, the power absorbed by the film is assumed to leave the film by convection and radiation with negligible heat flow along the film. (The results calculated for these ideal conditions represent a bound on the accuracy; heat flow along the film could cause additional error [15].)

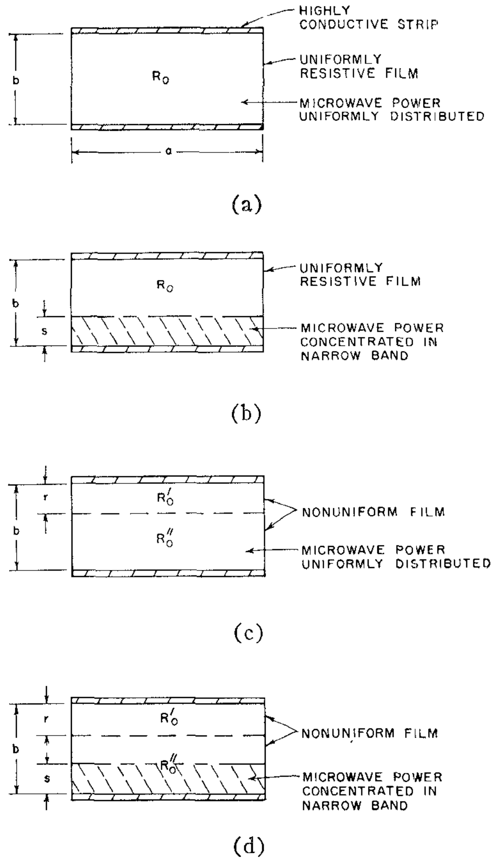


Fig. 11—Four rectangular films with different combinations of dc and microwave power distribution.

The four films illustrated carry the same dc bias current uniformly distributed over the film. Also, microwave power of w watts is being absorbed uniformly over the area of films shown in Fig. 11(a) and (c) and nonuniformly over the area of films shown in Fig. 11(b) and (d) where all the power is concentrated in the shaded area (such a power distribution is not encountered in practice, but the results to be derived would be the same). Finally, films shown in Fig. 11(a) and (b) have uniform resistivity over their areas and the total resistance of each film is R_0 after biasing, but before injection of microwave power; films shown in Fig. 11(c) and (d) are divided into two areas, each with different resistivities and with total biased resistances of R_0' and R_0'', respectively, where $R_0=R_0'+R_0''$. Thus equal amounts of dc and microwave power are being absorbed by each of the four films (but in different distributions of the dc and microwave powers in each case). We now ask how

[5] Averaged over measurements using various films and bolometer mounts at several frequencies.

the biased resistance R_0 of each film has changed because of the absorption of microwave power in the described manner (Fig. 11). We assume that all films are made of the same metal and therefore have similar temperature coefficients of resistivity as well as power coefficients of temperature (temperature rise per absorbed power density), and that actual temperature changes are very small. If we find that the resistances of all films are equal after absorption of microwave power, then the power-measuring equipment, which is sensitive to resistance changes, will correctly indicate that equal amounts of microwave power were absorbed by each film. Otherwise, unequal final resistance values will indicate (incorrectly) that different amounts of microwave power were absorbed by the films.

We first define the quantities

Δ = temperature coefficient of resistivity (1/°C)

τ = power coefficient of temperature (°C/watt-cm²).

For film shown in Fig. 11(a) in Fig. 7 with w watts of microwave power absorbed uniformly over the film we find that the resistance is

$$R_{1a} = R_0\left(1 + \frac{w\tau\Delta}{ab}\right). \tag{1}$$

For film shown in Fig. 11(b) with w watts of microwave power absorbed by the shaded region of height s we find the total resistance is

$$R_{1b} = \frac{b-s}{b} R_0 + \frac{s}{b} R_0\left(1 + \frac{w\tau\Delta}{as}\right) \tag{2}$$

which can be simplified to

$$R_{1b} = R_0\left(1 + \frac{w\tau\Delta}{ab}\right) = R_{1a},$$

the same result as (1). Film shown in Fig. 11(c) is analyzed in a similar manner,

$$R_{1c} = R_0'\left[1 + \frac{\left(\frac{r}{b}\right) w\tau\Delta}{ar}\right] + R_0''\left[1 + \frac{\left(\frac{b-r}{b}\right) w\tau\Delta}{(b-r)a}\right] \tag{3}$$

which again reduces to

$$R_{1c} = R_0\left(1 + \frac{w\tau\Delta}{ab}\right) = R_{1a}.$$

In the case of film shown in Fig. 11(d), where both the film resistance and microwave power are nonuniformly distributed, we have for $s<b-r$,

$$R_{1d} = R_0' + R_0''\frac{b-s-r}{b-r} + R_0''\frac{s}{b-r}\left(1 + \frac{w\tau\Delta}{as}\right) \tag{4}$$

which after simplification becomes

$$\left.\begin{aligned} R_{1d} &= R_0' + R_0''\left[1 + \frac{w\tau\Delta}{a(b-r)}\right] \\ &= R_0\left[1 + \frac{R_0''}{R_0}\,\frac{w\tau\Delta}{a(b-r)}\right] \end{aligned}\right\}\ \text{Case of film shown in Fig. 11(d).}$$

$$\neq R_{1a}$$

Eq. (5) is different from (1), the result for the three previously analyzed films. We conclude that in general a different amount of dc substitution power is required to balance the bridge in the last case than in the three earlier cases. Now note that nonuniform dc current will result in nonuniform film resistivity. By analogy, any film in which both the RF and dc powers are nonuniformly distributed, such as a rhombic film, will behave in a manner similar to that of film shown in Fig. 11(d). The rhombic films that were tested did indeed show a greater error in power measurement, in line with the above reasoning.

V. Conclusions

Thin-film bolometers that completely cover the waveguide cross section have been shown to be useful for measuring the combined power of at least two waveguide modes, and the power in three modes separately, over a very broad frequency range. It may be expected that these bolometers will measure power accurately up to frequencies for which the film thickness is less than the skin depth. These devices are therefore ideal for obtaining total power measurements when the mode distributions and frequency spectrums are unknown.

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Five Layer Optical Maser Amplification

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Summary—**The optical maser is treated in the manner of a Fabry-Perot resonator with an active medium. Five layers are considered: air, reflector, active medium (ruby), reflector, and air. General equations are derived using the method of boundary value problems in which it is assumed that incident coherent radiation falls normally on the surface. It is suggested that the presence of lossless one-quarter wavelength reflectors will enhance the amplification of the device in that less pumping may be required for a given length of ruby. The role of the reflectors in oscillation conditions is shown to be of importance. Methods are indicated for the calculation of amplitude and phase for an idealized amplifier.**

INTRODUCTION

CONSIDERABLE work has been reported on a three layer optical maser amplifier consisting of air, ruby and air,[1] treating the system as a transmission line, or boundary value problem in electromagnetic theory. Both approaches lead to the same equations for amplification and oscillation. Furthermore, these equations are equivalent to those developed by V. N. Smiley[2] for the same boundary conditions (system composed of air, ruby, air). In the latter work the analysis was based on a Fabry-Perot structure containing an active medium. In the following analysis, the calculations are carried further in that the multiple internal reflections in the reflectors are considered together with the internal reflections in the active medium. The optical maser amplifier is conceived as a five layer structure consisting of air, reflector, active material such as ruby, reflector, and air. General expressions are developed to represent transmitted gain in such a system. The special case is then examined in which the reflectors are lossless and are one-quarter wavelength in thickness. This system is then reduced to an equivalent three layer structure. Numerical calculations predict that for reflector materials with high dielectric constants and negligible loss, the length of the crystal, or the pumping power required for a given amplification, can be substantially reduced. One might have thought this possible since the quarter wavelength reflectors increase multiple internal reflections which in turn provide a superposition of waves resulting in greater power output. However, other factors enter the situation. A critical length is calculated for a given negative attenuation which produces maximum gain or oscillations. At lower values of length the gain in transmission increases with length. As the length increases beyond the critical value, the transmitted gain decreases.

Calculations are also made on the phase shift of the transmitted electric field compared with incident elec-

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